CS459/698 Privacy, Cryptography, Network and Data Security

A pinch of Homomorphic Encryption

Fall 2024, Tuesday/Thursday 02:30pm-03:50pm

What is Homomorphic Encryption?

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- **Definition:** Homomorphic encryption is a cryptographic technique that allows computations to be performed on encrypted data without requiring decryption.
- Raw data can remain fully encrypted while it's being processed, manipulated, and run through various algorithms.

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- Raw data can remain fully encrypted while it's being processed, manipulated, and run through various algorithms.
- Idealized in 1978, fully realized in 2009 by Craig Gentry





Homomorphic Encryption for Dummies

"Anybody can come and they can stick their hands inside the gloves and manipulate what's inside the locked box. They can't pull it out, but they can manipulate it; they can process it... Then they finish and the person with the secret key has to come and open it up and only they can extract the finished product out of there." -- Craig Gentry

Computing on Ciphertexts (Simple Math)



https://chain.link/education-hub/homomorphic-encryption

Computing on Ciphertexts (More sophisticated math)



Homomorphic Encryption in the Wild



Government Sectors:

FHE streamlines the often cumbersome processes that were traditionally required to maintain the confidentiality of investigations. Our innovative Zero Footprint Investigations solution is revolutionizing the way government agencies handle sensitive data related to investigations. This solution enables agencies to keep the subjects of their investigations completely confidential while still accessing and analyzing crucial data sources.



Financial Services:

By leveraging FHE's capabilities, financial institutions can enhance their fraud detection and prevention efforts by tapping into data they originally would not have access to. This innovative approach not only strengthens security measures but also fosters global cooperation in combating financial crimes. Healthcare Industry:

FHE provides a secure framework for sharing and analyzing sensitive medical data, addressing challenges related to privacy and data protection. The global pandemic in 2020 underscored the pressing need for enhanced collaboration among healthcare researchers and organizations.



FHE empowers information service providers and data brokers to eliminate the need for large-scale data transfers by enabling secure computations on encrypted data, thus streamlining interactions with government and public entities.

https://dualitytech.com/blog/homomorphic-encryption-making-it-real/

Output

Homomorphic Encryption in the Wild

- Used as a tool in many real-world scenarios:
 - O <u>https://www.ibm.com/security/services/homomorphic-encryption</u>
 - O <u>https://www.statcan.gc.ca/en/data-science/network/homomorphic-encryption</u>
 - <u>https://www.statcan.gc.ca/en/data-science/network/statistical-analysis-homomorphic-encryption</u>
 - <u>https://www.intel.com/content/www/us/en/developer/tools/homomorphic-encryption/overview.html</u>
 - O <u>https://www.microsoft.com/en-us/research/project/microsoft-seal/</u>
 - https://machinelearning.apple.com/research/homomorphic-encryption
 - https://github.com/apple/swift-homomorphic-encryption
 - O https://github.com/google/heir
 - O https://github.com/google/fully-homomorphic-encryption?tab=readme-ov-file

E.g., Homomorphic Encryption for Secure Voting

• Microsoft's ElectionGuard

Microsoft Microsoft On the Issues Our Company V News and Stories V Topics V More V

Protecting democratic elections through secure, verifiable voting

May 6, 2019 | Tom Burt - Corporate Vice President, Customer Security & Trust

What does ElectionGuard do?

ElectionGuard is a way of checking election results are accurate, and that votes have not been altered, suppressed or tampered with in any way. Individual voters can see that their vote has been accurately recorded, and their choice has been correctly added to the final tally. Anyone who wishes to monitor the election can check all votes have been correctly tallied to produce an accurate and fair result.

So what is this all about?

Homomorphic Encryption

Consider the following:

Two ciphertexts use the same key, $\mathbf{c} = E_K(\mathbf{x})$, $\mathbf{d} = E_K(\mathbf{y})$ Let **f()** be a function that operates over plaintext **x** and **y**

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g() is a homomorphic function on the ciphertexts c, d, ...

Partial versus Fully Homomorphic Encryption

The function on the plaintexts is:

...either multiplication or addition **but not both**.



...either multiplication or addition, **or both**



Homomorphic Encryption Types				
	Partially	Somewhat	Leveled Fully	Fully
Rating	Simple	Intermediate	Advanced	Most advanced
Computations	Addition or multiplication	Addition and / or multiplication	Complex but limited	Complex and unlimited
Use cases	Sum or product	Basic statistical analysis	AI/ML, MPC	AI/ML, MPC

Only useful for **simpler** operations. Relatively **efficient**.

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https://chain.link/education-hub/homomorphic-encryption

of operations that can be performed is **bounded** and the accuracy of the computation may **degrade** as more operations are performed.

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	Can perform a has a pre-defi	Can perform an arbitrary # of computations on encrypted data, if it has a pre-defined set of computations specified ahead of time .			
	Partially	Somewhat	Leveled Fully	Fully	
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	Enables any # of computations to be performed on encrypted data without a predefined sequence or limit . Computationally expensive .			
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A partial homomorphic encryption scheme based on El Gamal

Recap: ElGamal Public Key Cryptosystem

- Let **p** be a prime such that the DLP in $(\mathbf{Z}_{p}^{*,})$ is infeasible
- Let α be a generator in \mathbf{Z}_{p}^{*} and "a" a secret value
- $\mathbf{Pub}_{\mathbf{K}} = \{(p, \alpha, \beta): \beta \equiv \alpha^{a} \pmod{p}\}$
- For message "m" and secret random "k" in Z_{p-1} : • $e_K(m,k) = (y_1, y_2)$, where $y_1 = \alpha^k \mod p$ and $y_2 = m\beta^k \mod p$

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• For y_1, y_2 in Z_p^* :

 $\bigcirc \quad d_{K}(y_{1}, y_{2}) = y_{2}(y_{1}^{a})^{-1} \mod p$

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Public key is p, α, β

Bob's Pub_K \rightarrow (p, α , β) $y_1 \equiv \alpha^k \pmod{p}$ Bob's Priv_K \rightarrow a $y_2 \equiv m \beta^k \pmod{p}$ $\beta \equiv \alpha^a \pmod{p}$





Bob's Pub_K \rightarrow (p, α , β) $\mathbf{y}_1 \equiv \alpha^k \pmod{p}$ Bob's Priv_K $\rightarrow \mathbf{a}$ $\mathbf{y}_2 \equiv m \ \beta^k \pmod{p}$ $\beta \equiv \alpha^a \pmod{p}$

Consider Multiplicative HE



Goal: show how the multiplication of ciphertexts corresponds to the multiplication of plaintexts.

Bob's Pub_K \rightarrow (p, α , β) $y_1 \equiv \alpha^k \pmod{p}$ Bob's Priv_K \rightarrow a $y_2 \equiv m \beta^k \pmod{p}$ $\beta \equiv \alpha^a \pmod{p}$





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Consider Additive HE

Multiplicative: The math of ElGamal ensures that multiplying the encrypted values corresponds to multiplying the original plaintext values.

Additive: Here, we no longer have the same nice properties of how exponents play together.

- **"Crazy" idea:** Something like $g(E_K(\alpha^x), E_K(\alpha^y)) = E_A(\alpha^{x+y})$ could work
 - But we would need to break the discrete log of α^{x+y} to retrieve the sum
 - Only really works for small x and y

The Paillier Partially Homomorphic Encryption Scheme

- Proposed by Pascal Pailler in 1999
- The Paillier cryptosystem is a public-key cryptosystem known for its additive homomorphic properties.
- The security of the Paillier cryptosystem is based on the difficulty of the composite residuosity class problem
 - This problem involves determining whether a given number is an *n*-th residue modulo *n*² for a composite *n*.

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- Ciphertexts are mod N^2

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g is a generator

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- Ciphertexts are mod N^2
- Choose r ; plaintext m (mod p) is encrypted as g^m r^N (mod N²)

From the product of ciphertexts to addition of plaintexts

• Multiply encryption of **m**₁ and **m**₂:

$$\begin{split} & \mathsf{E}(\mathsf{m}_1,\mathsf{r}_1) \cdot \mathsf{E}(\mathsf{m}_2,\mathsf{r}_2) \bmod \mathsf{N}^2 = \\ & \mathsf{g}^{\mathsf{m}1} \cdot \mathsf{g}^{\mathsf{m}2} \cdot \mathsf{r}_1^{\mathsf{N}} \cdot \mathsf{r}_2^{\mathsf{N}} \bmod \mathsf{N}^2 = \\ & \mathsf{g}^{\mathsf{m}1+\mathsf{m}2} \cdot (\mathsf{r}_1 \cdot \mathsf{r}_2)^{\mathsf{N}} \bmod \mathsf{N}^2 \end{split}$$

• Multiply encryption of **m**₁ and **m**₂:

 $E(m_1,r_1) \cdot E(m_2,r_2) \mod N^2 =$ $g^{m1} \cdot g^{m2} \cdot r_1^N \cdot r_2^N \mod N^2 =$ $g^{m1+m2} \cdot (r_1 \cdot r_2)^N \mod N^2$

If factorization of *N* is known, breaking the DL is efficient
⇒ Efficient additive HE, even for large numbers
Paillier's Encryption Scheme

• Multiply encryption of **m**₁ and **m**₂:

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• If factorization of **N** is known, breaking the DL is efficient \Rightarrow Efficient additive HE, even for large numbers

$$D(E(m_1,r_1)\cdot E(m_2,r_2) mod n^2) = m_1 + m_2 mod n_1$$

Simplica Numara!

DGHV: A Fully Homomorphic Encryption Scheme

Fully Homomorphic Encryption (FHE)

- Many schemes now, usually abbreviated by the first letters of the last names of the authors
- Different security assumptions (not factoring or discrete log)
 Lattice problems: Learning with errors, ...

Examples:

- First construction by Gentry in 2009
- E.g. FV, BGV, or DGHV (not used in practice)

The **DGHV** Fully Homomorphic Encryption Scheme

- FHE scheme whose security is based on the difficulty of the approximate greatest common divisor (AGCD) problem.
 - Finding the greatest common divisor of a set of integers that are close to multiples of a secret integer.

Fully Homomorphic Encryption over the Integers

Marten van Dijk MIT Craig Gentry Shai Halevi IBM Research IBM Research Vinod Vaikuntanathan IBM Research

June 8, 2010

https://medium.com/@j248360/explaining-the-dghv-encryption-scheme-1acb6cd74dd6 https://www.esat.kuleuven.be/cosic/blog/co6gc-homomorphic-encryption-part-1-computing-with-secrets/ https://github.com/coron/fhe

Consider Simplified DGHV (not used in practice)

- *m* ∈ {0, 1}
- Secret key: prime **p**

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- *m* ∈ {0, 1}
- Secret key: prime **p**
- Encryption
 - Choose q, r such that r is random noise
 - $\circ c = q.p + 2.r + m$

Consider Simplified DGHV (not used in practice)

- *m* ∈ {0, 1}
- Secret key: prime **p**

Encryption

- Choose q, r such that r is random noise
- $\circ c = q.p + 2.r + m$
- Decryption
 m = *c* mod 2 ⊕ ([*c*/*p*] mod 2)

Computing with Simplified DGHV

• Ciphertexts

$$\circ c_1 = q_{1.}p + 2.r_1 + m_1$$

 $\circ c_2 = q_2p + 2.r_2 + m_2$

Computing with Simplified DGHV

• Ciphertexts

$$\circ c_1 = q_{1.}p + 2.r_1 + m_1$$

 $\circ c_2 = q_{2.}p + 2.r_2 + m_2$

Addition

$$\circ c_1 + c_2 = (q_1+q_2).p + 2.(r_1+r_2) + m_1 + m_2$$

Note that noise grows linearly

Computing with Simplified DGHV

• Ciphertexts

$$\circ c_1 = q_{1.}p + 2.r_1 + m_1$$

 $\circ c_2 = q_{2.}p + 2.r_2 + m_2$

Addition

$$\circ c_1 + c_2 = (q_1 + q_2).p + 2.(r_1 + r_2) + m_1 + m_2$$

Multiplication

$$c_1 \cdot c_2 = q'.p + 2.r' + m_1.m_2$$

 $c_1 \cdot c_2 = q'.p + 2.r' + m_1.m_2$
 $c_1 \cdot c_2 = q'.p + q_1.m_2 + q_2.m_1$

Note the increased growth of the noise. (no longer linear). One gets a new ciphertext with noise **roughly twice larger** than in the original ciphertexts c1 and c2.

The bootstrapping problem in FHE

Bootstrapping... in Fully HE Schemes

- If $r > p/2 \Rightarrow$ decryption fails on DGHV
 - Also a problem for other schemes.
- If the noise **grows too much**, it can **corrupt** the encrypted data and make it unusable
- Each operation increases the noise, so one must control this growth

Bootstrapping... in Fully HE Schemes

- To obtain a FHE scheme, (i.e. unlimited addition and multiplication on ciphertexts), one must **reduce** the amount of noise in a ciphertext
- **Bootstrapping** is a procedure that reduces noise to it's initial lenght
 - Still, bootstrapping is <u>slow</u> in most fully HE schemes
 - Thus, w/ fully HE, aim to <u>avoid</u> subsequent multiplications
 - DGHV does not have bootstrapping



Practical FHE Schemes

• FV, BGV, BFV, CKKS

- Lattice-based encryption schemes
- Encrypt vectors (usually as polynomials)

• TFHE

Fully HE over the Torus
Usually encrypts bits
Very fast bootstrapping (frequently performed)
https://tfhe.github.io/tfhe/

Try it... on your own 😳

- Download Microsoft's SEAL library and hack away!
 - <u>https://www.microsoft.com/en-us/research/project/microsoft-seal/</u>
 Create a key
 - Encrypt two 8 bit numbers bit-wise using batch encoding (allows rotation)
 - Perform comparison, for each position: If prefix is equal and bits are different, output 1 if bit of first number is 1; else output 0
 - $^{\bigcirc}$ Decrypt result



Encrypted Search Algorithms

Tradeoffs: Efficiency vs. Security



Cryptographic Mechanisms



Examples of Data Structures

• Dictionaries map labels to values



• Get: DX[w₃] returns id₂

• Multi-maps map labels to tuples



• Get: MM[w₃] returns (id₂, id₄)

Examples of Data Structures



Examples of Data Structures



Trusted Client





Untrusted Server







Untrusted Server









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How do we model leakage?

 The "Baseline" leakage profile for response-revealing EMMs

 \checkmark (L_S, L_Q, L_U) = (dsize, (qeq, rid), usize)

- The "Baseline" leakage profile for response-hiding EMMs
 ✓ (L_S, L₀, L₁₁) = (dsize, geg, usize)
- There exists several new constructions with better leakage profiles
 - AZL and FZL [Kamara-Moataz-Ohirimenko'18]
 VHL and AVHL [Kamara-Moataz'19]

	Information			
	intermetteri			
Response Length	D(q)			
Query Equality	$\boldsymbol{q}_i = \boldsymbol{q}_j$			
Co-Occurrence	$ D(q_i) \cap D(q_j) $			
Response Identity	$\{i: D_i \in q(D)\}$			
Response Volume	$\{ D_i _b: D_i \in q(D)\}$			

(Simplified)

Leakage Attacks Types



Leakage Attacks against ESAs



Leakage Attacks against ESAs



ESA Techniques Overview

Technique	Leakage	Query Time	
Fully Homomorphic Encryption (FHE)	• None	Linear	Considered secure but inefficient
Oblivious RAM (ORAM)	 Response Length + Volume 	Sublinear	Our work
Structured Encryption (STE)	 Query Equality Response Identities + Volumes 	Optimal	Considered efficient and Has some leakage
Property-Preserving Encryption (PPE)	 Ciphertext Equality Ciphertext Order All STE leakage 	Optimal	Considered efficient but provides low level of security [NKW15]

Uncertainty Of Security



Uncertainty Of Security



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Encrypted Search Algorithms: Real-World Deployments.





[Always Encrypt '15]

- Encrypted Relational Database (ERDs)
- Property-Preserving Encryption (PPE)



[Client-Side Field Level Encryption'19]

- Encrypted Non-Relational Database (EnRDs)
- Property-Preserving Encryption (PPE)

Queryable Encryption'23

- Encrypted Non-Relational Database (EnRDs)
- Structured Encryption (STE)

aws

[Document Encryption'23]

- Encrypted Non-Relational Database (EnRDs)
- Property-Preserving Encryption (PPE)
A Few Announcements

Assignment 3 is <u>due today</u> 4pm

 $\odot\,$ No-penalty late policy period until Saturday 4pm



• Student Course Perceptions – Available until Dec 3

O https://perceptions.uwaterloo.ca/

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