CS489/689
Privacy, Cryptography, Network and Data Security

PIR, SSE, Blockchains, Bonus Applied Crypto

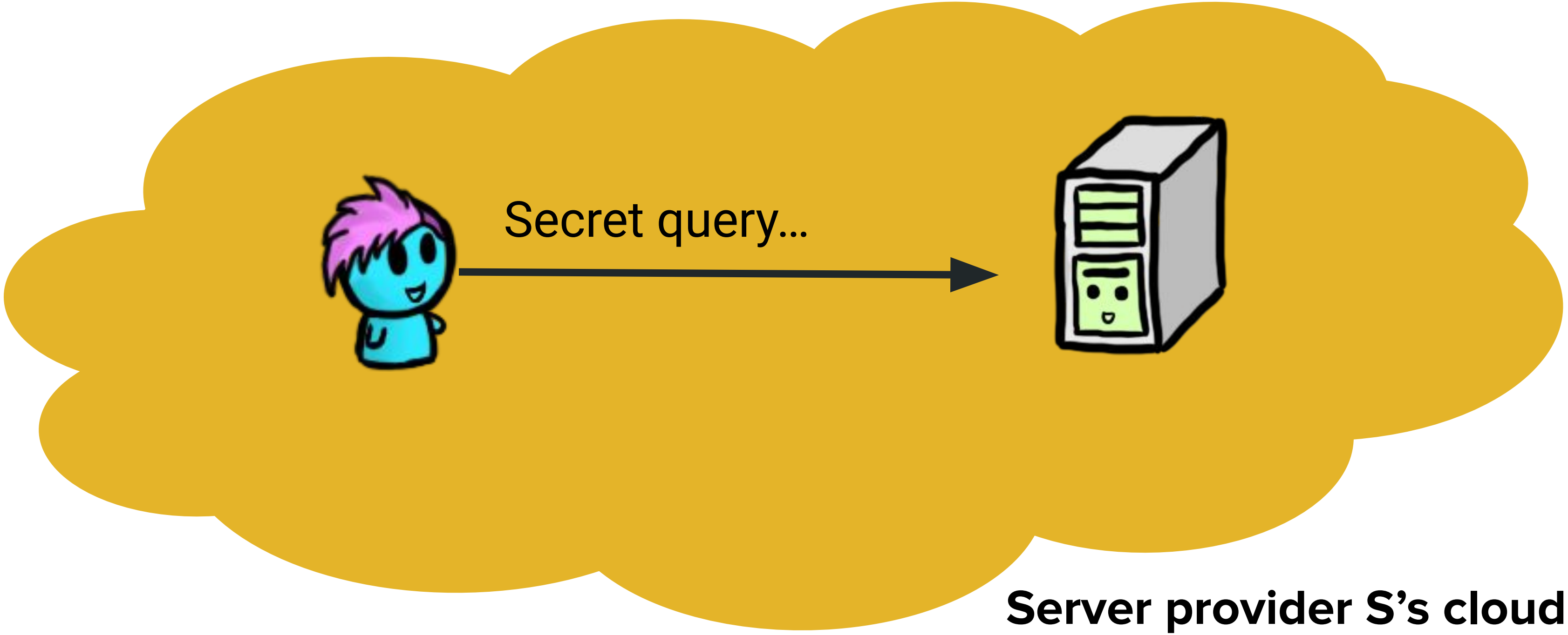
Winter 2023, Tuesday/Thursday 8:30-9:50am

Last two classes!

- Private data access (PIR, SSE)
- Blockchains
- Applied cryptography

Private Database Queries

Introduction...



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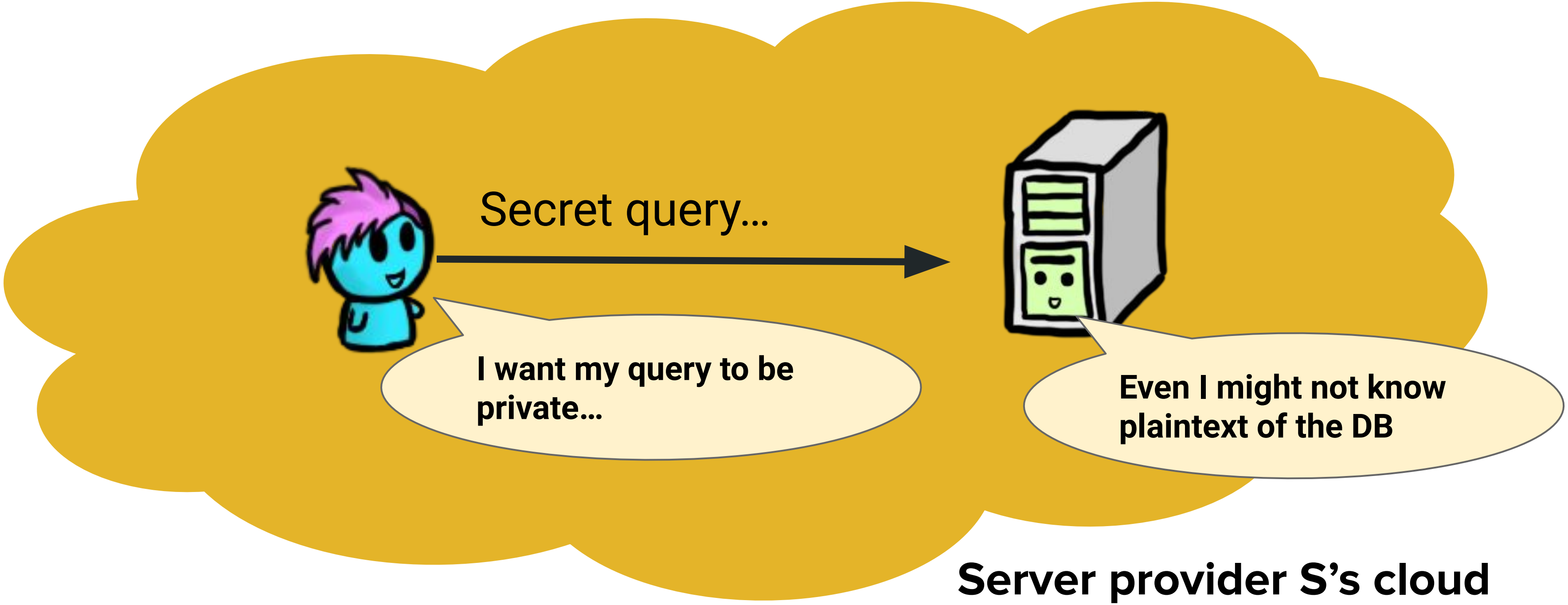

Real-world Example

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- The client wants to check whether the password they just created for an Internet site is in that database
- If it is, they should not use it
- If it is not but revealed to the database, the password should not be used either anymore
- Hence, the client needs to query **without revealing the password**

Classification, or Terms for Private Queries

- Server learns matching elements / does not learn matching elements
  - Searchable encryption / Private information retrieval (PIR)

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  - Searchable encryption / Private information retrieval (PIR)
- Keyword / index query
  - Keyword PIR / PIR

PIR

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Carol the client has index i

PIR

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Server has DB $d_1, \ldots, d_n$

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Goal Correctness: Client learns d_i

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PIR

Carol the client has index i

Server has DB $d_1,\ldots,d_n$

Goal Correctness: Client learns d_i

Goal Security: Server does not learn index i

Catch: There is no security requirement that client only learns d_i

Complexity of PIR

Act.

- Theorem: The server's search complexity for a single query i is O(n)
- Proof:

Q: Why?

Complexity of PIR

- Theorem: The server's search complexity for a single query i is O(n)
- Proof:
  - By contradiction
  - Assume the server does not access (consider) item j for query i
  - The server has just learnt that $i \neq j$

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Caveat: This applies for a <u>single query i</u> but not a set of concurrent queries

Information-Theoretic PIR

- Assume two servers S_1 and S_2 with the same database D

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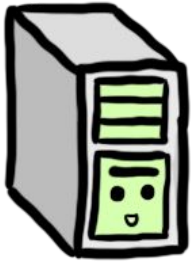
- The client computes $d_i = s_1 + s_2$

Notes on IT-PIR

- Requires at least two servers (who cannot collude)
- Communication complexity O(n)

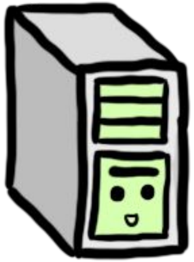
Q: Why?

Computational PIR



Assume a single server

Computational PIR



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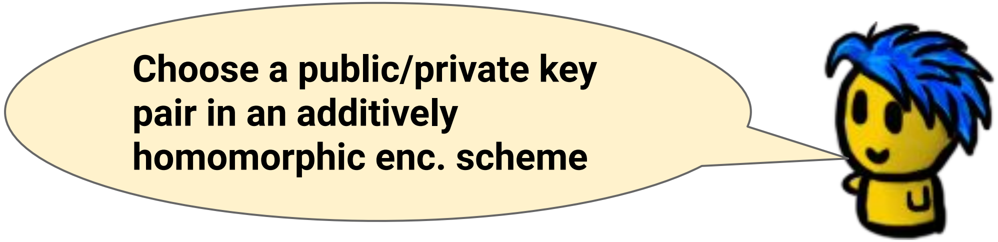
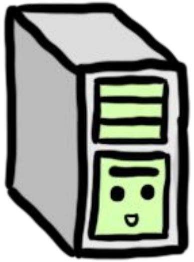


Computational PIR



Assume a single server

Choose a public/private key pair in an additively homomorphic enc. scheme

- Let I be the indication vector
- $C \rightarrow S$: $E_C(I(j))$ (for $1 \le j \le n$)
- Server computes $c = \prod_{1 \le j \le n} E_C(I(j))^\wedge(d_j) = E_C(\Sigma_{1 \le j \le n} I(j)\ d_j)$
- $S \rightarrow C$: c
- Client decrypts $d_i = D_C(c)$

Hashing Elements…or Searches?

- Let w be the query keyword
- Let there be a hash function h, such that h(w) = i
- Dealing with collisions:

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  - Query may return constant number of elements as a block (note public database assumption)
  - If D is updated, so may be h

Server computes indicator vector

- Assume fully HE
- Let w_k be the k-th bit of w and $d_{j,k}$ the k-th bit of d_j $(1 \leq k \leq u)$
- $C \longrightarrow S$: $E_C(w_k)$
- For each d_j
  - The server computes $E_C(I(j)) = \prod_{1 \leq k \leq u} \neg(E_C(w_k) \oplus E(d_{j,k}))$
- Caveat: Computation cost is high

Repeated keywords

- So far, each keyword was unique
- How can we deal with repeated keywords?
  - The indicator vector has now multiple 1 entries
  - The size of result set that can be returned in one query is fixed

Searchable Encryption

Searchable Encryption

- Server does learn the matching entries for the query
- Server does not know database

Upload

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- Client sorts { t_k } and uploads sorted table T

SSE Query

- Let w be the query keyword
- $C \rightarrow S$: $v = H_K(w)$
- Server computes $t_k = H(v||k)$ $(1 \leq k \leq n)$
- Server returns entries matching t_k in T

Notes on searchable encryption

- One round protocol, independent of result set size
- Server computation complexity $O(\max_j(c_j) \log n)$
- Client communication complexity O(1)
- Server communication complexity $O(\max_j(c_j))$
- Much more efficient than PIR

Efficiency comes at a price

- Server learns which elements match the query

⇒ Same elements, same query

- Assume the server knows which is the most common keyword
  - Abstractly, the server has background knowledge about the frequency of keywords
- Assume the server knows which query is “popular”
  - Abstractly, the server has background knowledge about the frequency of queries
- Etc.

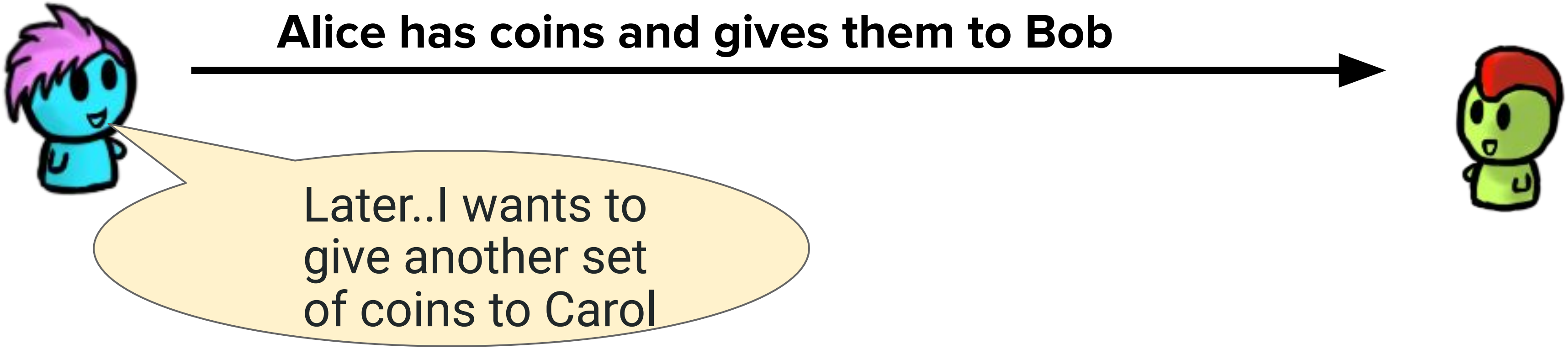
Blockchain

Introduction

- There is a large and changing number of parties
- There is some state S that all parties share
  - S could be the balance of an account/set of transactions (ledger)
  - S could be the state of a state machine (smart contract)

The double spending problem

The double spending problem



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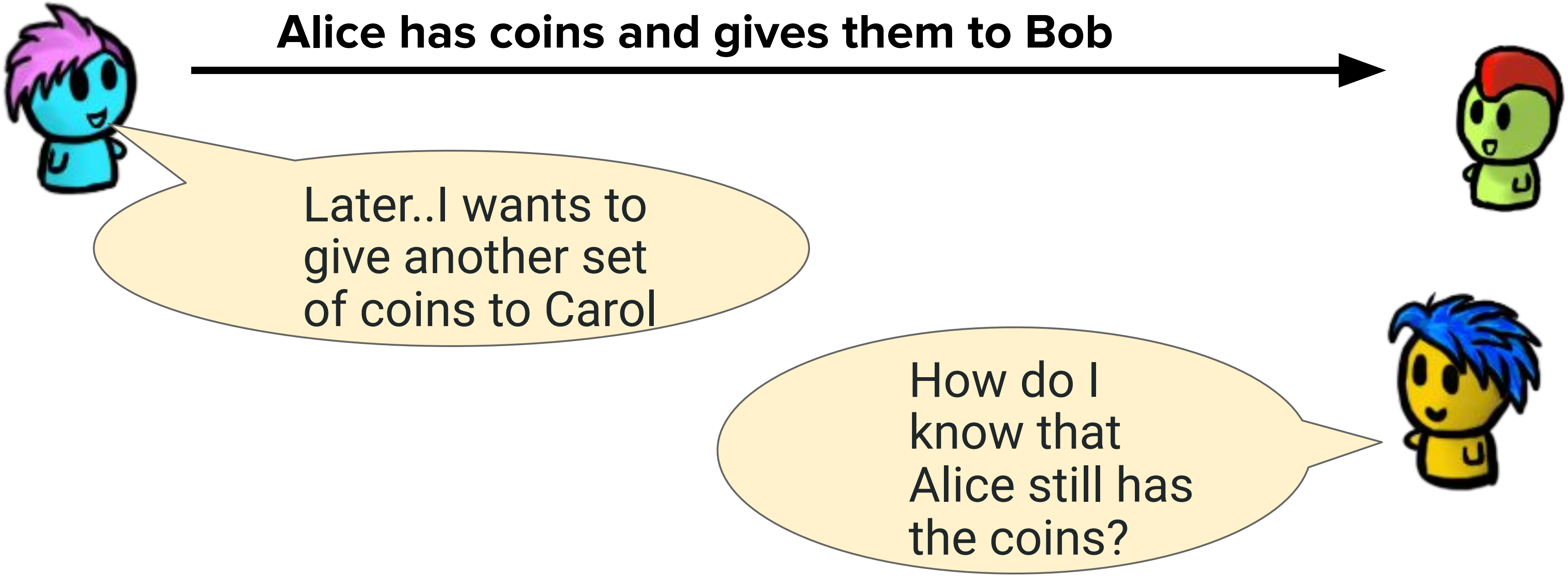

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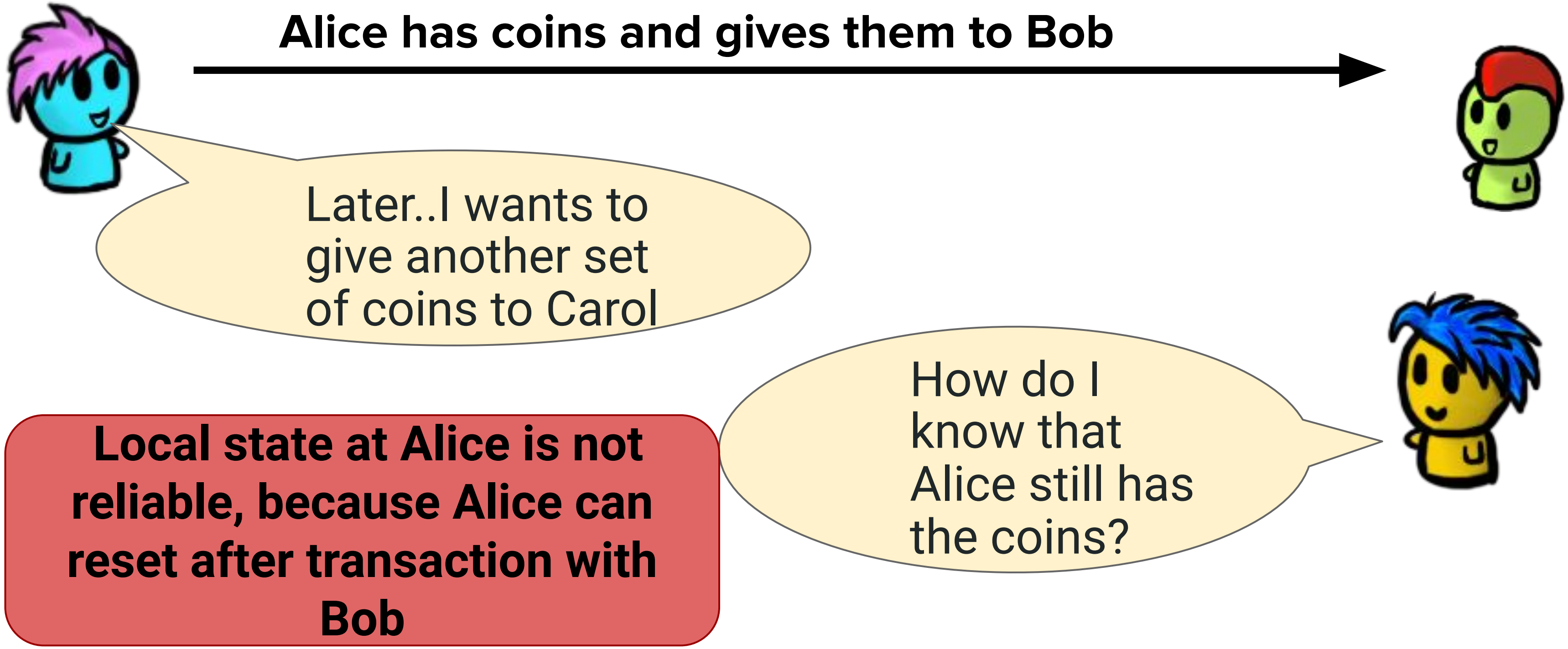

The double spending problem

Alice has coins and gives the

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of c

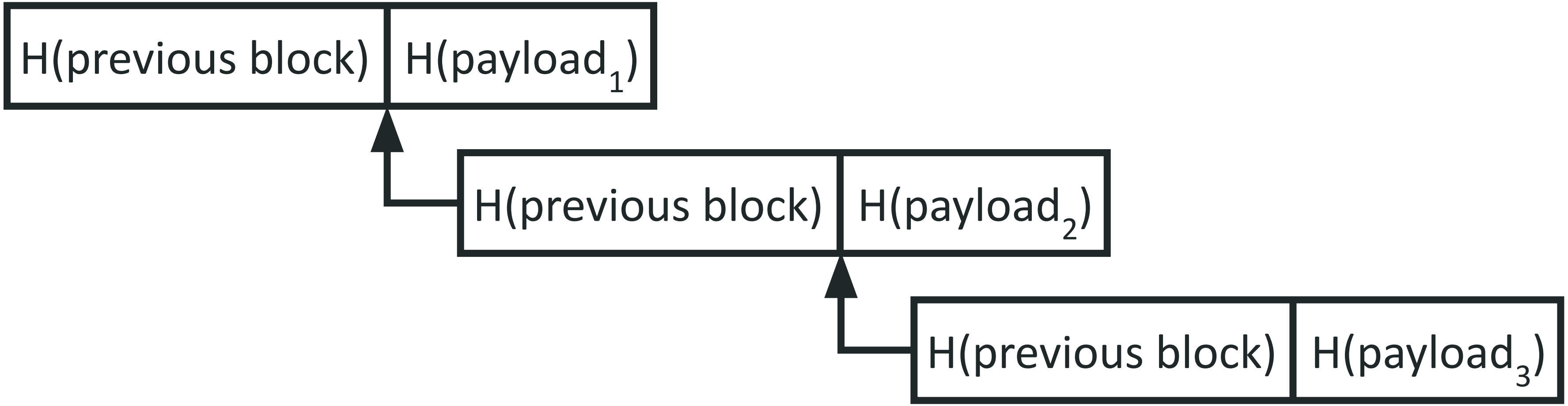
Thus: need a global state
E.g., Trusted party (bank)
Distributed (blockchain)

How do I know that Alice still has the coins?

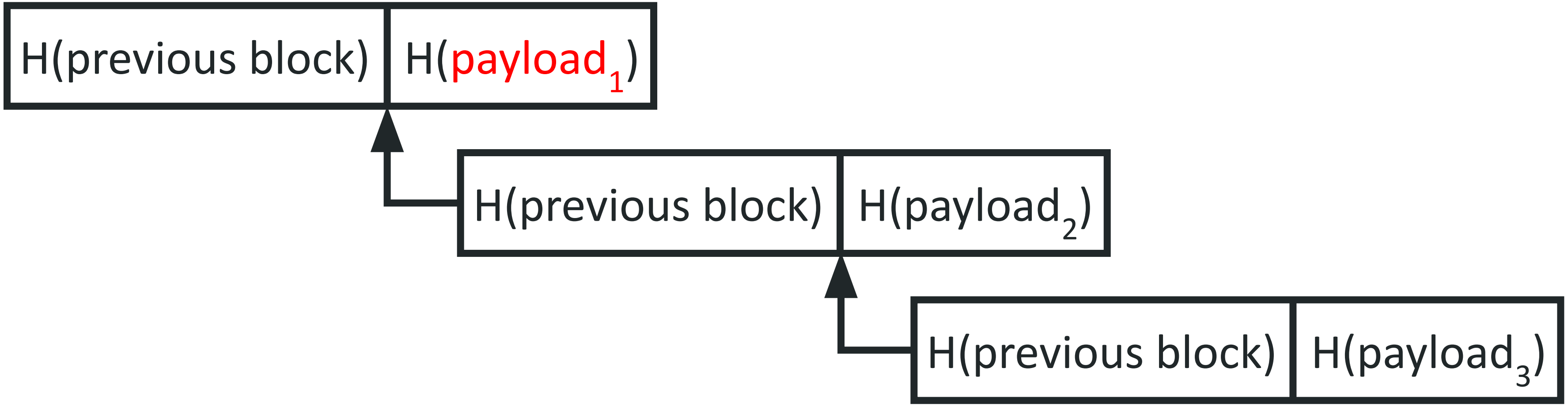
Local state at Alice is not reliable, because Alice can reset after transaction with Bob

What is the state?

- The state grows (append only): $payload_1$ || $payload_2$ || $payload_3$



State is append-only



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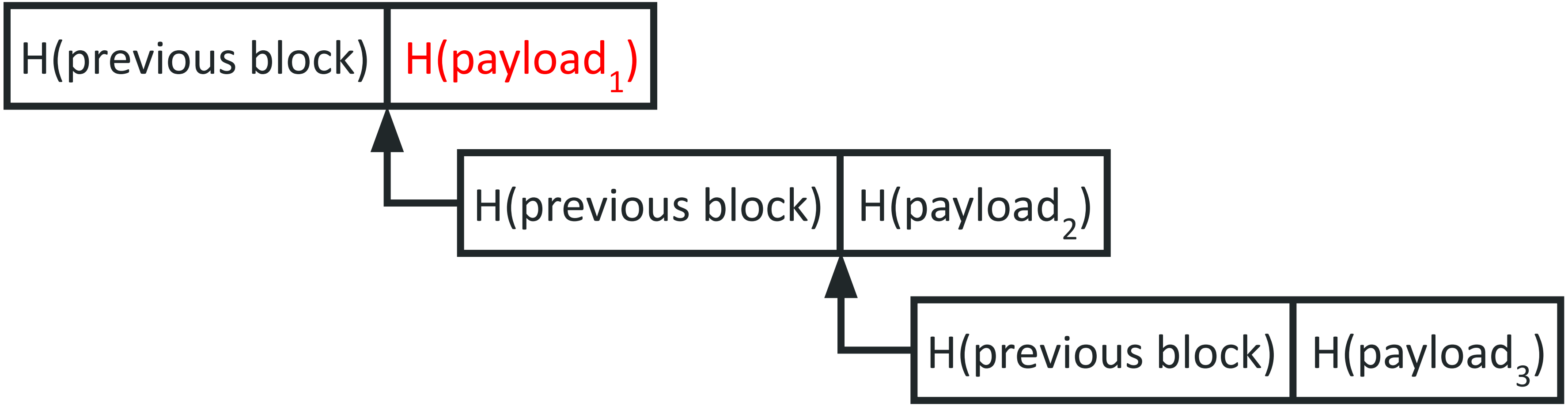

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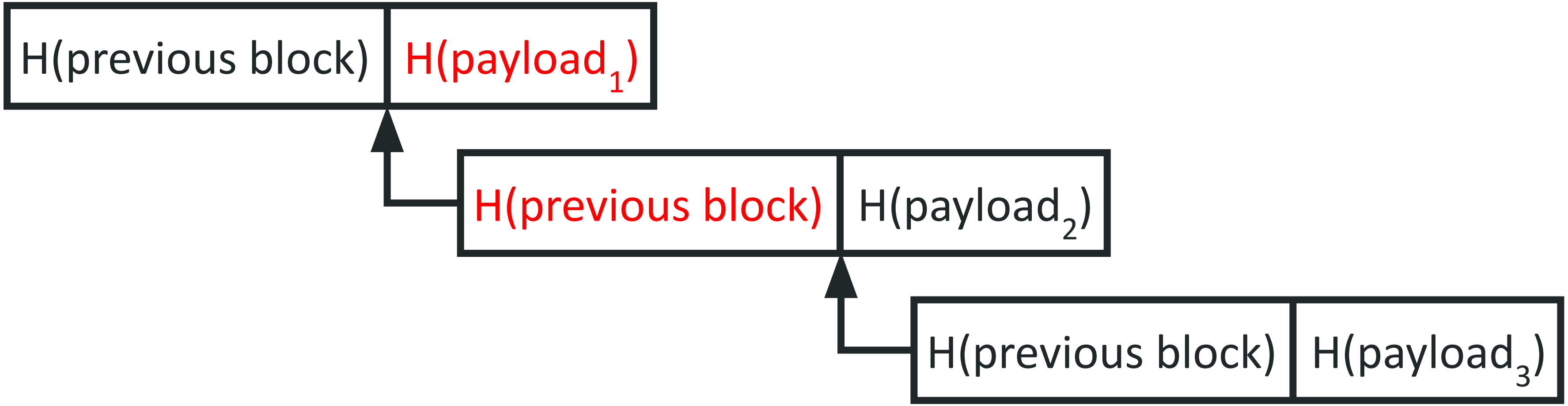

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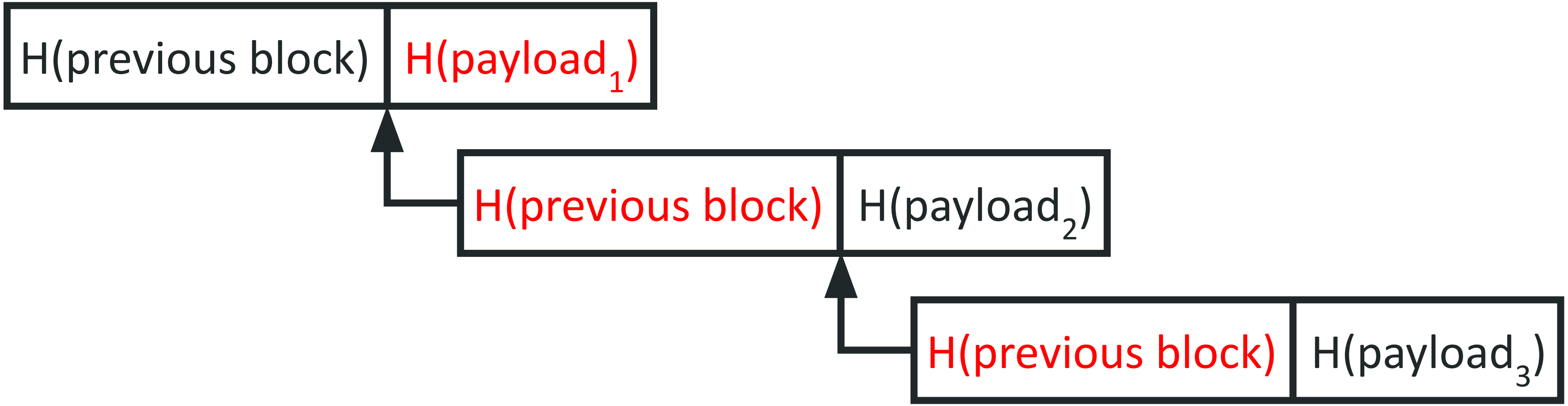


Who can add to the state?

- Key Idea
  - Not everyone can add to the state, but only selected one/few
- Permissioned block
  - Fixed set of signers
- Permissionless block chain
  - Leader election
    - Proof of Work
    - Proof of Stake

Permissioned Blockchain

- Step 1: Signers agree on next state
  - Consensus protocol
    - CS454
- Step 2: Joint signature of block

H_1(previous block)	H_2(payload)	Signature($H_1 \| \| H_2$)

  - Multi-party computation
    - Threshold signature

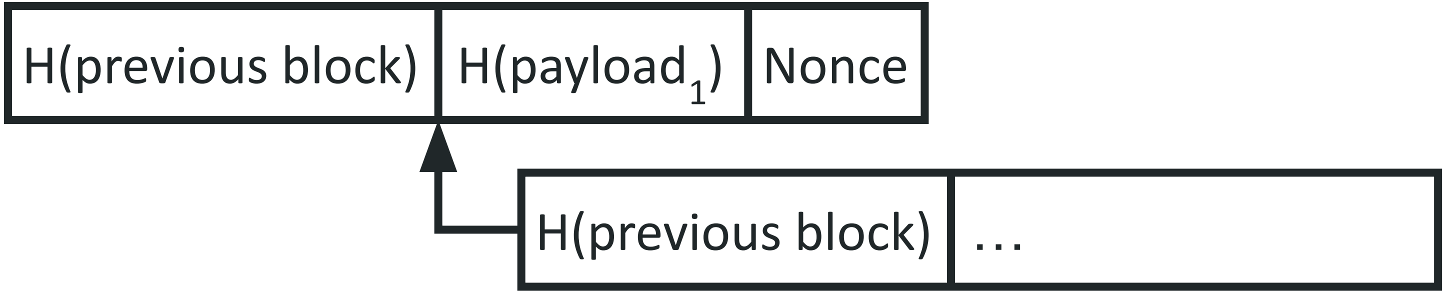
Threshold signature

- Distribution
  - KeyGen: ($pubKey$, $privKey_1$, $privKey_2$, …)
  - $Signer_i$ gets $privKey_i$
- Signature
  - $signature_i$ = Sign(message, $privKey_i$)
  - signature = Reconstruct($signature_1$, $signature_2$, …)
    - At least t out of n: (t, n)-threshold signature
- Verification
  - Verify(signature, message, pubKey) = T / ⊥

Permissionless Blockchain

- Anybody should be able to become signer
  - Open system
- Needs to qualify
  - Computation: Proof-of-Work
  - State-dependent: Proof-of-Stake

Proof of Work



- Requirement:
  - H(previous block) < threshold · No. hash values
    - Starts with leading zeros
- H is a one-way function
  - Need to try different nonce until requirement is fulfilled

Expected number of trials

- Output of hash function is uniform in [0, No. hash values – 1]
- Probabilty of success
  - Pr[H < threshold · No. hash values] = threshold
- Expected number of trials
  - E[H | H < threshold · No. hash values] = 1 / threshold

How to determine threshold

- Constant expected time
  - Bitcoin
- Global threshold:
  - threshold = prev. thres. · (2016 · 10 minutes) / (time since 2016th last block)
- Adjusts to the number of parties in the system (miners)

What if the miner is malicious?

- In permissionless blockchain, step 1 (consensus) of permissioned blockchain is missing
- There is no guarantee that the miner will use agreed payload

Nakamoto Consensus

- Assume other (honest) parties somehow agree on “valid” state
- They will ignore block and continue with previous state

$\Rightarrow$ There can be different chain ends in the system

What is the agreed state

- If an honest miner agrees with more than one state, which one should they append to?
- Answer: The longest

Why?

- The longest chain has used the most work
- If majority of miners is honest, then the longest chain has used the most honest miners’ work
- Therefore if a majority is honest (> 50%), the longest chain will be appended the fastest
- Honest parties outrun malicious parties
  - What if the adversary controls >50% of the computing power?

Careful

- Honest parties not always in the lead
- Need to wait until payload has “settled”
- E.g. in Bitcoin, the recommendation is 6 blocks (60 minutes)

Caveat

- Computing hash values does not provide useful results
- Incentive is by transaction fee (a share of each transaction's value)
- Massive energy loss
  - As much as Argentina (May 2022, https://news.climate.columbia.edu/2022/05/04/cryptocurrency-energy/)
  - 0,5% of all electricity consumed in the world (September 2021, https://www.nytimes.com/interactive/2021/09/03/climate/bitcoin-carbon-footprint-electricity.html)
  - Daily tracking: https://ccaf.io/cbeci/index

Proof of Stake

- Stake is a function of the state
  - E.g. account balance divided by total wealth
- Similar idea to mining
  - Signature (Verifiable Random Function)

  Sign(H(previous block), timestamp) < stake · No. VRF values

What if the miner is malicious?

- Honest miners can punish malicious miner
  - Burn stake (e.g. set account balance to 0)
- With multiple chains, how to determine which is the "valid" one?
  - Ethereum:
    - Punish forker (equivocator)
    - Vote on "valid" chain
      - Punish minority votes
- How to determine "valid" chain, when new to the system?
  - Ethereum: Trusted nodes broadcast last hash

What is in a state?

- Transactions
  - Sender public key
  - Recipient public key
  - Amount
  - Signature by sender public key
- Smart contract
  - Sender public key
  - Opcodes (program)
  - Storage

How to validate transaction?

- Check account balance (from blockchain state)
- Verify signature

Smart contract execution

- Read old storage / program (of blockchain state)
- Execute program
  - Use volatile memory (stack)
- Write updated storage

How to validate smart contract execution?

- Need to execute program and verify updated storage
- What if the program does not hold (or takes very long)?
  - Limit number of computational steps
    - Charge per step
  - Stop if limit has been exceeded
    - Ethereum: Gas limit

Block Chain and Private Data?

Block Chain and Private Data?

Caveat: Blockchain compatibility with sensitive data?

Other exercises

Exercise

- Download data sets D_1, D_2 from XXX
- Encrypt each keyword in D_1 with searchable encryption
- Use D_2 to attack D_1
  - Query for each keyword in $D_1 \cap D_2$
  - Observe the matching entries in D_1 (server's view)
  - Use D_2 to guess the keywords in D_1

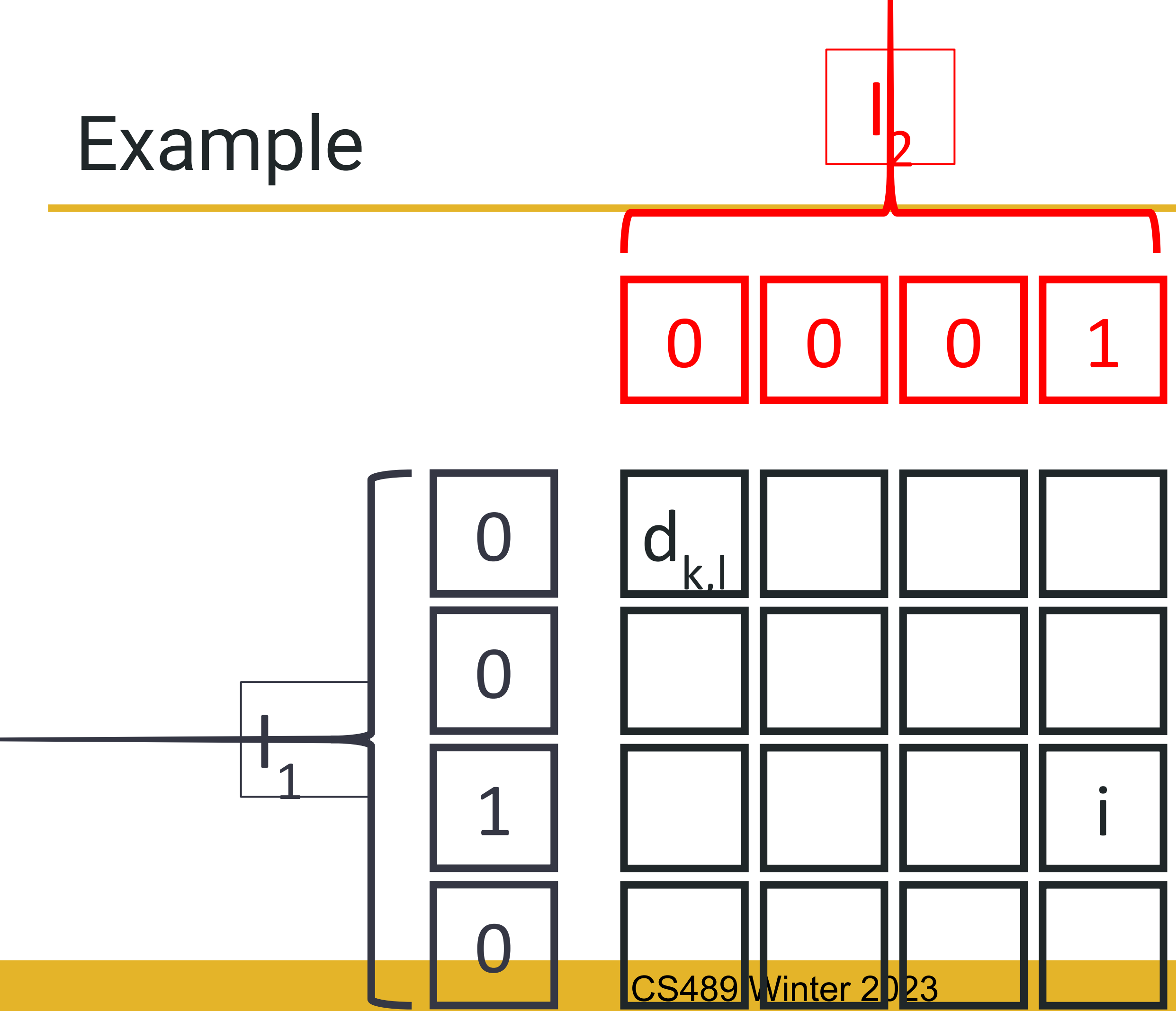
Notes

- So far communication complexity O(n)
- We can do better in C-PIR

Folding

- Server arranges database into a f-dimensional hypercube (we consider f = 2 square)
  - $d_{k,l} = d_i$ ($i = k\sqrt{n} + l$)
- Clients generates f (two) indicator vectors I_1 and I_2
  - $I_1(k) = 1$, $I_2(l) = 1$ (else 0)
- Server computes $c_l = \prod_{1 \le j \le \sqrt{n}} E_C(I1(j)) \wedge (d_{j,l})$ (for $1 \le l \le \sqrt{n}$)

Example



Folding II

- If $E_C()$ is fully homomorphic
- Server computes
  - $e = \sum_{1 \le j \le \sqrt{n}} E_C(I_2(j)) \cdot c_{j,m}$
- Client decrypts decrypts $e = D_C(e)$

Folding III

- If $E_C()$ is additively homomorphic
- Server splits c_l into m "chunks" $c_{l,m}$
  - Note that Paillier ciphertexts are twice as long as their plaintexts
- Server computes
  - $e_m = \Pi_{1 \le j \le \sqrt{n}} E_C(I_2(j)) \wedge (c_{j,m})$
- Client decrypts $c_{l,m} = D_C(e_m)$, reconstructs c_l, decrypts $d_i = D_C(c_l)$

Exercise

- What is the communication complexity of C-PIR for f=2?
- What is the communication complexity of C-PIR for any f?
- What is the lower bound of the communication complexity of folding for C-PIR?

Keyword Queries

- Index queries: Dense, each index returns an element
- Keyword queries: Sparse, very few keywords return an element

$\Rightarrow$ Indicator vector becomes large compared to database size

Notes on C-PIR

- Computational cost of HE is very high
- Additively HE overhead is higher than fully HE overhead
- Additively HE overhead is so high, that downloading the entire database is faster

Exercise

- Create smart contract to accept mined nonce for
  - H(“University of Waterloo rules” || nonce)
  - With at most 2^{20} hashes
- https://ethereum.org/en/developers/

Returning the first result

- Let I be the indicator vector of length n
- Compute prefix $P_1 = \sum_{1 \le k \le j} I(k)$ (for $1 \le j \le n$)
- Compare prefix P_1 to 1^n: $I' = (P_1(j) - 1)$^$(p-1) \bmod p$
- Compute prefix $P_2 = \sum_{1 \le k \le j} I'(k)$ (for $1 \le j \le n$)
- Compare prefix P_2 to 1^n: $I'' = P_2(j) - 1)$^$(p-1) \bmod p$
- Caveat: Using clever math this can be optimized to one comparison

Example

I	0	1	0	1	1	1	0	0
P1	0	1	1	2	3	4	4	4
I’	0	1	1	0	0	0	0	0
P2	0	1	2	0	0	0	0	0
I’’	0	1	0	0	0	0	0	0