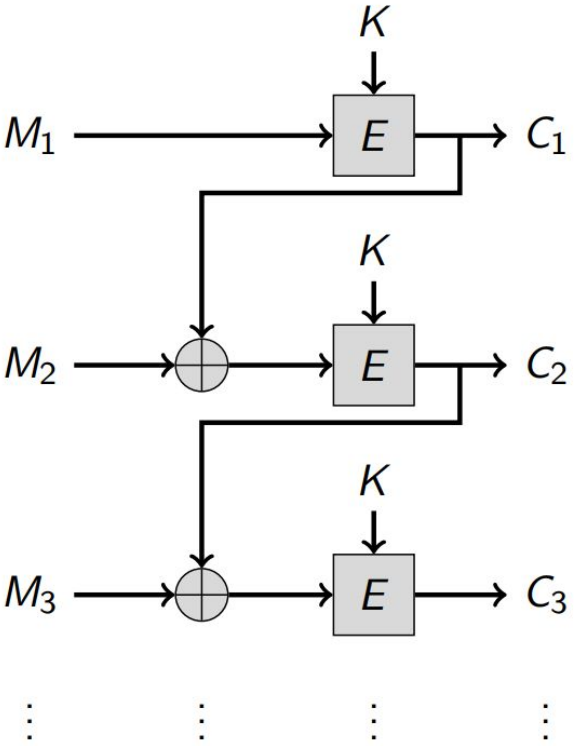
CS489/689
Privacy, Cryptography, Network and Data Security

Winter 2023, Tuesday/Thursday 8:30-9:50am

Assignment One

- Available on Learn
- Due **February 2nd 2023, 4pm**
- Written and programming
- Can currently do W1,2, and W3,4 after today
- Can do P1 now, P2 and P3, el gamal, Covered Jan 23

Clarification from Last Lecture



Q: Spot the difference?

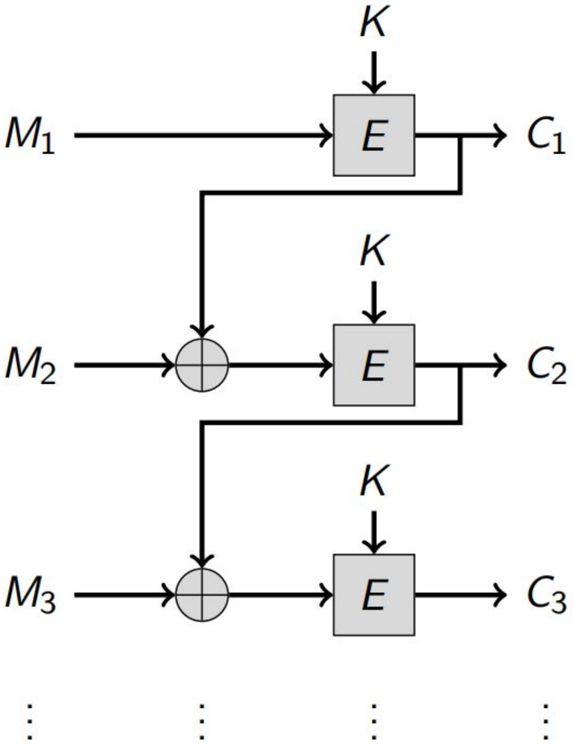
Q: Is it fixed this time?

Q: Does this avoid repeating patterns among blocks?

Q: What would happen if we encrypt the **message twice** with the **same key**?

A: $C_1 = E_K(M), C_2 = E_K(M) \Rightarrow C_1 = C_2$

Clarification from Last Lecture



- Let M and N be our messages
- Then we have m_1 m_2 and n_1 n_2
- m_1 m_2 produce ciphertext c_1 c_2
- n_1 n_2 produce ciphertext d_1 d_2
- If m=n, then c=d

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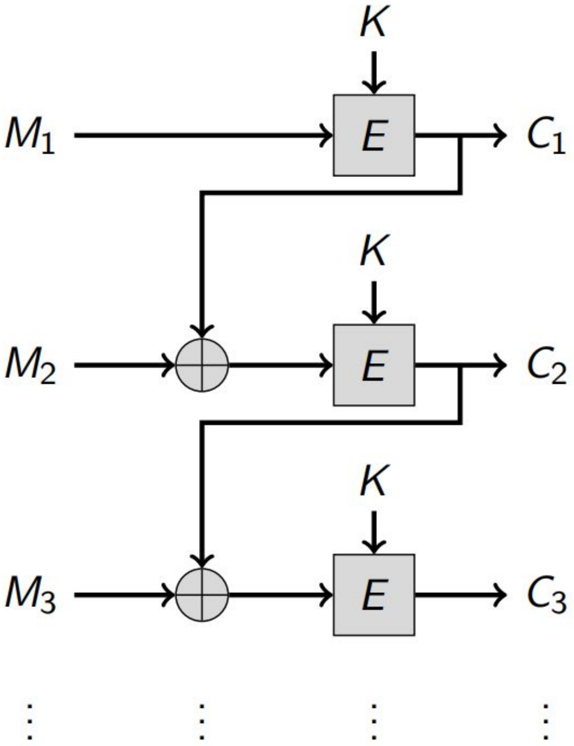

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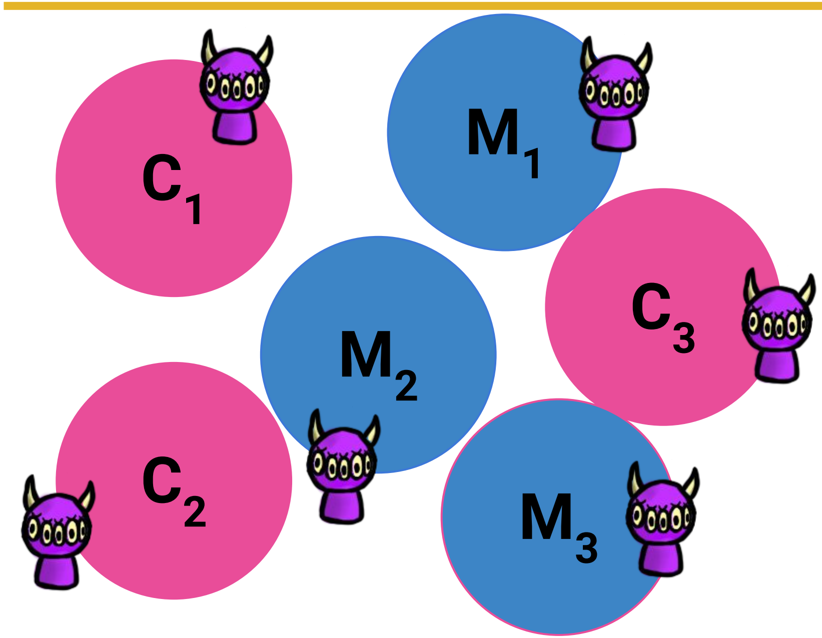
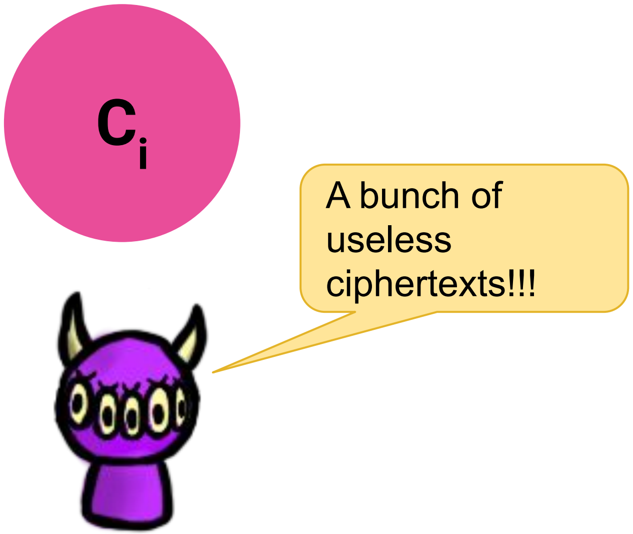
Helps facilitate ciphertext attacks as well as the general rule of patterns reveal information.

Cipher Security, IND-CCA2



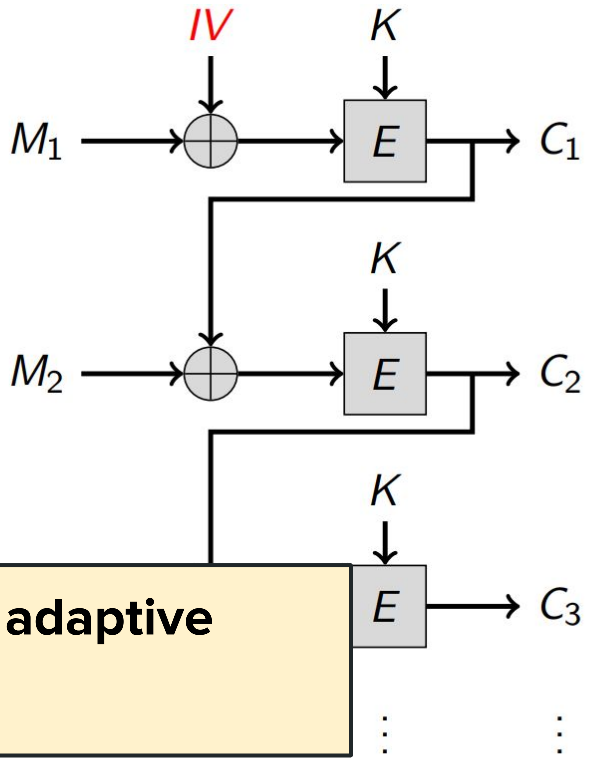
Adaptive chosen ciphertext attack



Eve cannot distinguish whether C_i is from M_1 or M_2

Recall CBC Mode for Block Ciphers:

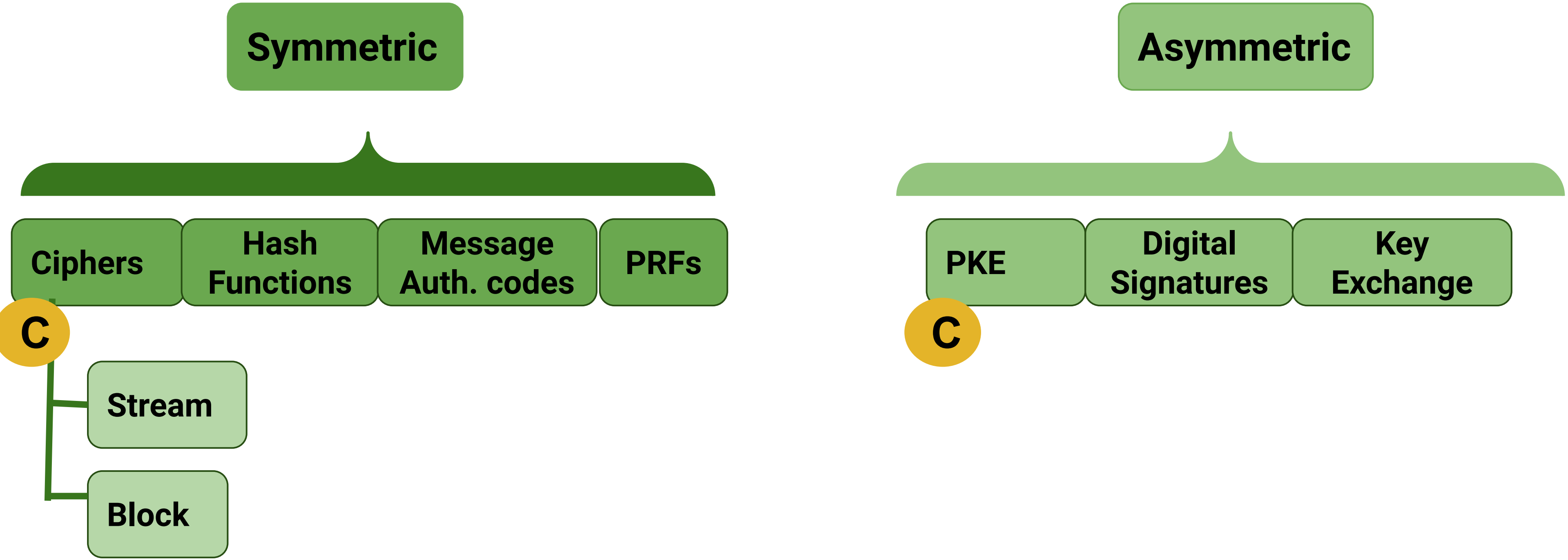
1. Generate a secret key k
2. Encrypt m using k and a generated IV
3. Decrypt c using k and the IV to get m



Security Goal: indistinguishability under adaptive chosen ciphertext attack (IND-CCA2)

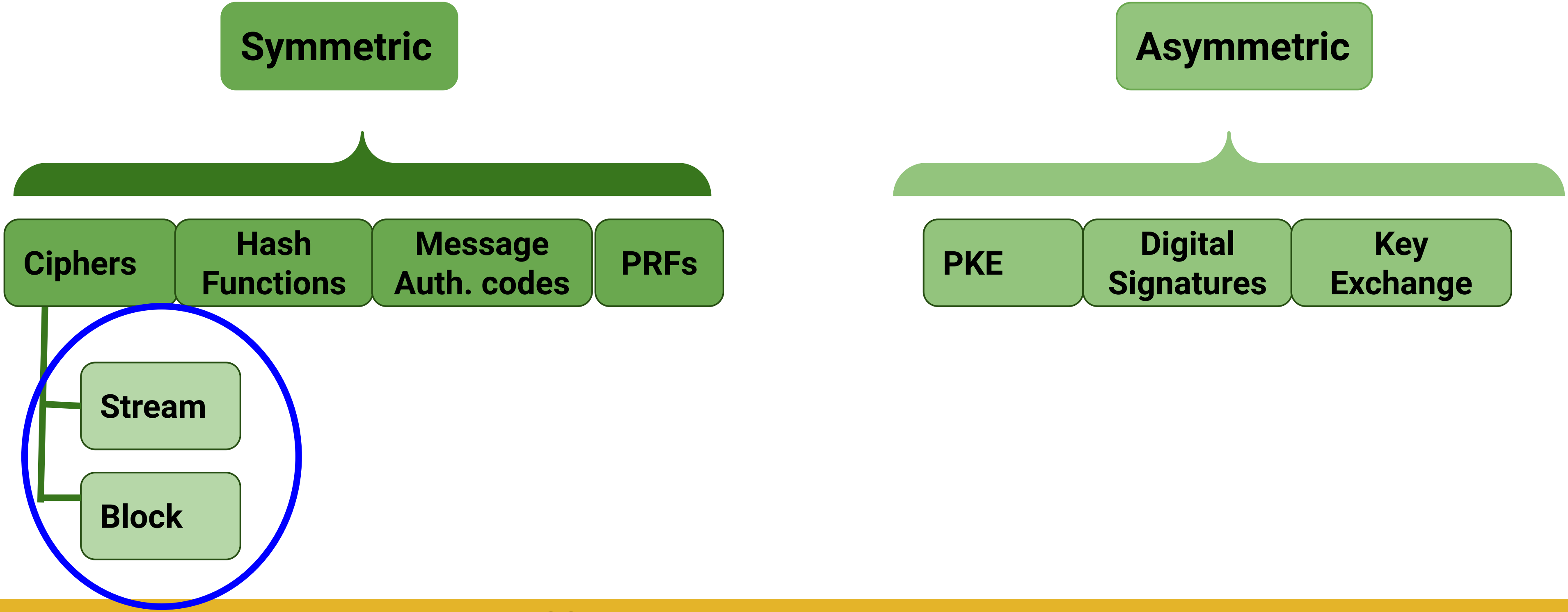
Today: More Cryptography

Cryptography Organization



Organization display source: Doug Stebila

Cryptography Organization



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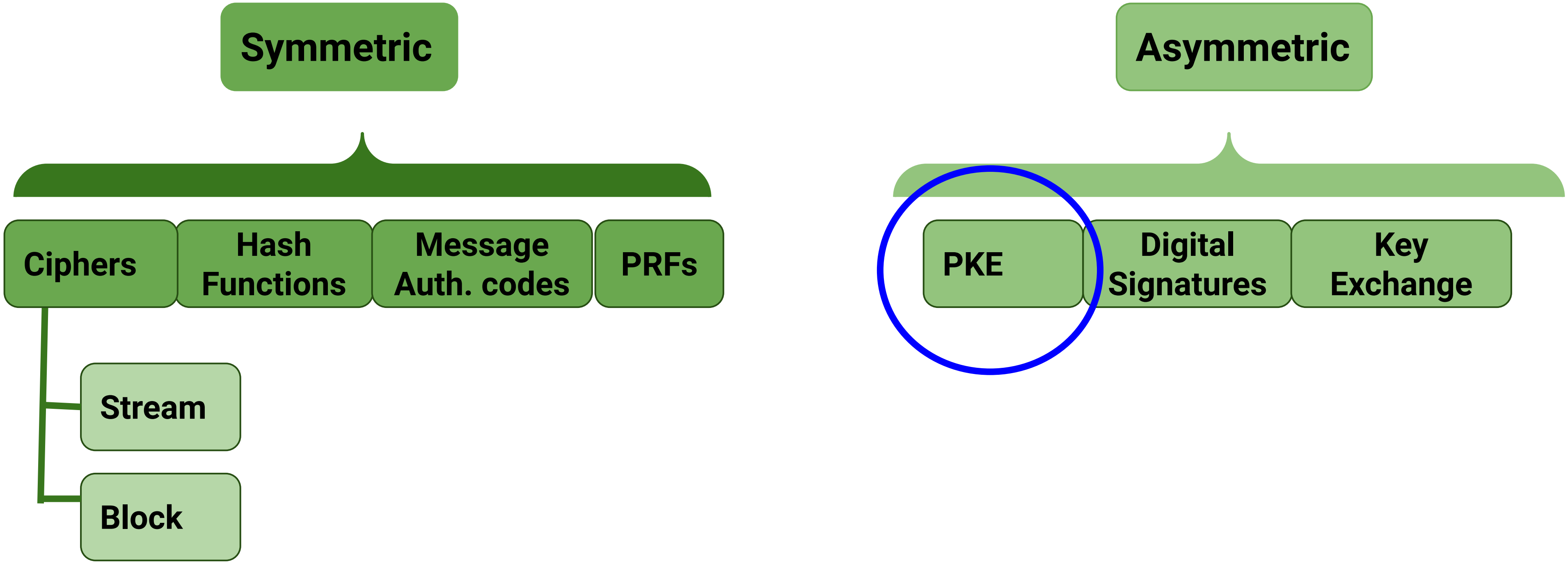
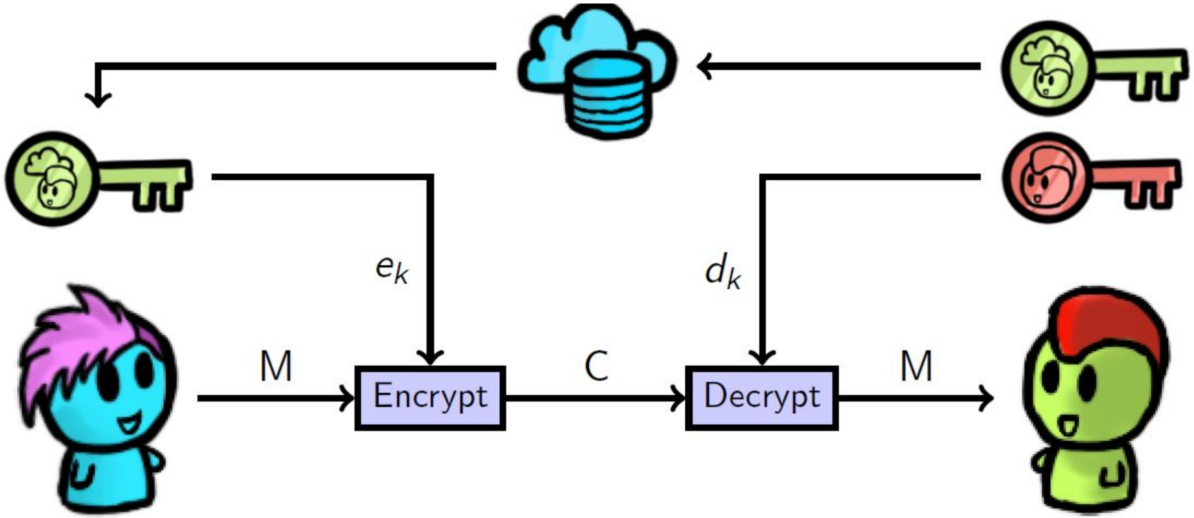


Public Key Cryptography, “1970s”

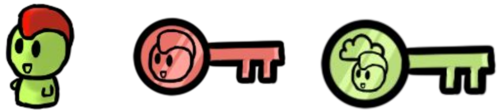


Examples:

- RSA, ElGamal, ECC, NTRU

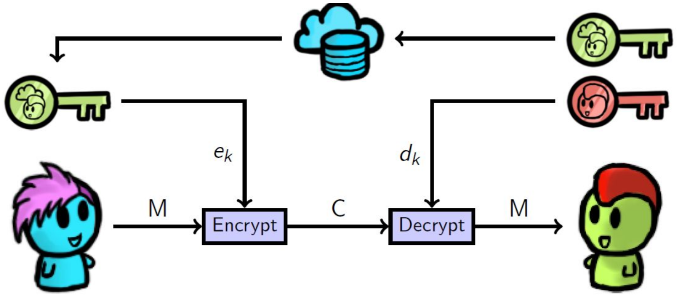
Steps for Public Key Cryptography?

1. Bob generates pair



2. Bob gives everyone the public key

3. Alice encrypts m and sends it
4. Bob decrypts using private key



5. Eve and Alice can't decrypt, only have encryption key

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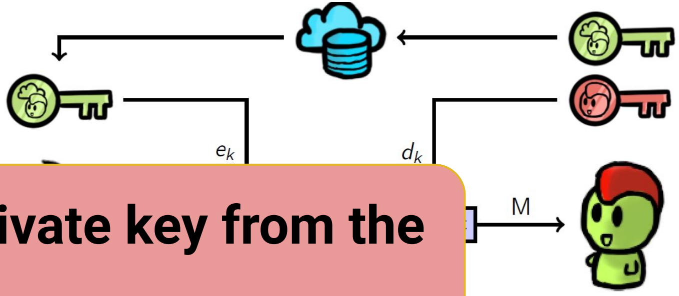

4. Bob d

It must be hard to derive the private key from the public key

5. Eve and Alice can’t decrypt, only have encryption key

Requirements for PKE

- The encryption function? Must be easy to compute 
- The inverse, decryption? Must be hard for anyone without the key  vs. 

Thus, we require so called “one-way” functions for this.

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Thus, we require so called “one-way” functions for this.

Because of encryption, also injective

Because of decryption, we need a “trapdoor”

Time for Textbook RSA

- Computational difficulty of the **factoring problem**
  - Given two large primes n = p*q, it is very hard to factor n.
- Modular arithmetic: integer numbers that “wrap around”
- Overview:

$$(m^e)^d \equiv m \pmod{n}$$

Easy for me to pick e, d, and n that satisfy that equation

Ugh. I know e and n (even m) and can’t find d!!!

Fun (?) Facts::

- RSA first popular public-key encryption method, published in 1977

Textbook RSA (Simplified Overview)

1. Choose two “**large primes**” *p* and *q* (secretly)
2. Compute n = p*q
3. “Choose” value e and find d such that $(m^e)^d \equiv m \pmod{n}$
4. **Public key**: (e, n)
5. **Private key**: d (other numbers tossed)
6. Encryption: $c \equiv m^e \pmod{n}$
7. Decryption: $c^d \pmod{n}$

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6. Encryp
7. Decryp

Decryption works, but factoring n breaks this!

Note:

- RSA? Rivest, Shamir, and Adleman

A Closer Look at RSA: Rec… Para…eters

1. Choose two **“large primes”**
2. Compute n = p*q
3. “Choose” valu…
4. **Public k…**
5. **Privat…** …)
6. Encryptio…
7. Decryp…n: c^d (r… n)

An “obvious” attack is for Eve
to attempt to factor *n*.
Then, compute c and e as
Bob would…

Note:
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A Closer Look at RSA: Rec Para eters

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WARNING: Factoring is
hard...but

Note:

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Factoring and RSA

- You want to factor the public modulus? 
- Good news, abundant literature on factoring algorithms
- Bad news, “appropriate” primes will not be defeated

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Bad primes: easily factored

Reference: Pollard p-1 Factoring Algorithm

Inputs: Odd integer n and a "bound" b

1. a = 2
2. for j =2 to B
   a. Do a = a^j mod n
3. d = gcd(a-1,n)
4. if 1 < d < n
   a. Then return (d)
   b. Else return ("failure")

Note:

- This algorithm dates from 1974

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Suppose p is a prime divisor of n and q <= B

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Then for every q|(p-1), (p-1)|B!

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- Well, $p|n$, so $a \equiv 2^{B!} \pmod{p}$
- $2^{p-1} \equiv 1 \pmod{p}$

Fermat's little theorem

1. $a = 2$
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- Well, p|n, so $a \equiv 2^{B!} \pmod{p}$
- $2^{p-1} \equiv 1 \pmod{p}$
- Since (p-1) | B!, it follows that …
- $a \equiv 1 \pmod{p}$ meaning p | (a-1)

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- Well, $p|n$, so $a \equiv 2^{B!} \pmod{p}$
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Since $p|n$ and $p|d$ with $d = \gcd(a-1, n)$, then d is a non-trivial divisor of n

Note:

- Assumes a<>1

Bad Primes: Pollard p-1 Factoring Example

- N = 15770708441, apply algorithm with B=180
- Found a = 11620221425 and computed d = 135979

$$15770708441 = 135979 * 115979$$

- The algorithm works because of **135979**

What are the factors of 135978?

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- By taking B>=173, 135978|B! as desired!

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4. **Public key**: (e, n)
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7. Decryp...

...**toring n breaks this!**

WARNING: this was textbook RSA, do **not** use!!!

Why not "Textbook RSA"? Start with an Example

Example: (Tiny RSA), p=53, q=101, e=139, d=1459

Encryption: $c \equiv m^e \pmod{n}$, **Decryption:** c (n)

- Compute n
- Compute $C_1 = E_e(1011)$. Verify the decryption works
- Compute $C_2 = E_e(4)$. Verify the decryption works
- Compute $D_d(C_1 * C_2)$. What is happening…and why?

Note::

- The * here indicates multiplication/compute a product

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Note:

- The * he

A: The decryption is the product of the original plaintexts!!!

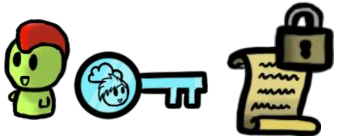
Malleability

$$\mathbf{A:}\ (m_1)^e * (m_2)^e = (m_1 * m_2)^e$$

It is possible to transform a ciphertext into another ciphertext that decrypts to a related plaintext

Undesirable (most of the time)

RSA and a Chosen Ciphertext Attack

- Alice is using RSA, public key (e, n) 
- Bob sends $c = E_e(m)$ 
- We are Eve! We snag c. 
- Alice…is confident about textbook RSA, will decrypt any ciphertext except c for us

Goal: Ask Alice to decrypt something (other than c) that helps us learn m

Executing CCA on Textbook RSA

- Alice is using RSA, public key (e, n)  
- Bob sends $c = E_e(m)$   
- We-Eve ask Alice to decrypt $c_2 = 2^e * c_1$ 

I am so clever mwahaha

Q: Decrypts to?

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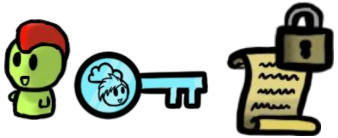
- Bob sends $c = E_e(m)$ 
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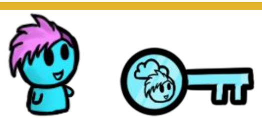
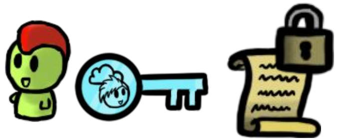
- Alice is using RSA, public key (e, n) 
- Bob sends $c = E_e(m)$ 
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Textbook RSA: vulnerable to CCA
Note: Can be addressed with padding techniques

Show Naive RSA Encryption is not IND-CPA Secure

1. Eve produces two plaintexts, m_0 and m_1 

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4. Sooo, Eve computes $c \leftarrow m_1^{\,e} \pmod N$

   If $c^* = c$ then Eve knows $m_b = m_1$
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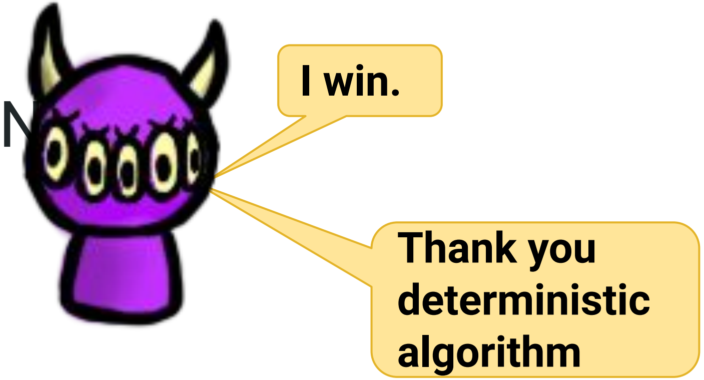


Implications of CCA versus CPA Security

Act.

Consider, when selecting an appropriate cryptosystem, what are the trade-offs (in security and practice) of a system that is:

- IND-CCA secure
- IND-CPA secure
- IND-CCA and IND-CPA secure

Identify: at least one implication for each of the above. Submit to Learn

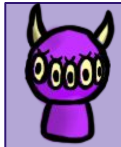
Adversaries and their Goals

You've assumed my goal is the secret/private key…

Adversaries and their Goals

Adversaries and their Goals

Goal 1: Total Break

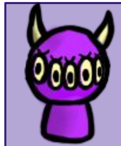


- Win the secret key k or
- Win Bob's private key k_b
- Can decrypt any c_i *for:*

$$c_i = E_k(m) \text{ or } c_i = E_{kb}(m)$$

- All messages using compromised k revealed
- Unless **detected** game over

Goal 2: Partial Break



- Decrypt **a ciphertext** *c* (without the key)
- Learn **some** specific information about a message *m* from *c*

**Need to occur with non-negligible probability.

- **Some (or a)** message revealed

Goal 3: Distinguishable Ciphertexts

- $P\{learn\ b \in \{0,1\}\}$ exceeds ½
- Distinguish between $E(m_1)$ and $E(m_2)$ or between E(m) and E(random string)

- The ciphertexts are leaking small/some information…

Semantic Security of RSA

- We saw CCA against Naive RSA

- We showed IND-CPA on Naive RSA

Show Naive RSA Encryption is not IND-CPA Secure

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Executing CCA on Textbook RSA

- Alice is using RSA, public key (e, n)
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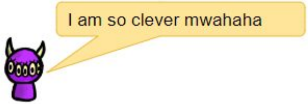

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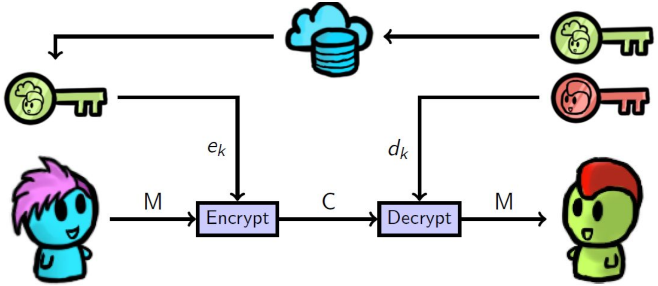
Fix it? Ciphertext Distinguishability

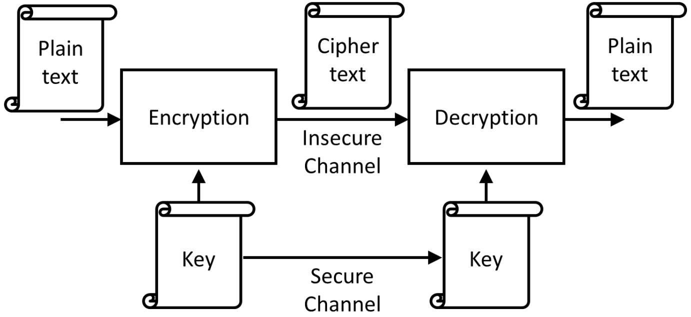
Goal: prove (given comp. assumptions) no information regarding the *m* is revealed in polynomial time by examining $c = E(m)$

- If E( ) is deterministic, fail
- Thus, require some randomization

RSA-OAEP: Optimal Asymmetric Encryption Padding

Practicality of Public-Key versus Private-Key





1. Longer keys
2. Slower
3. Different keys for E(m) and D(c)

1. Shorter keys
2. Faster
3. Same key for E(m) and D(c)

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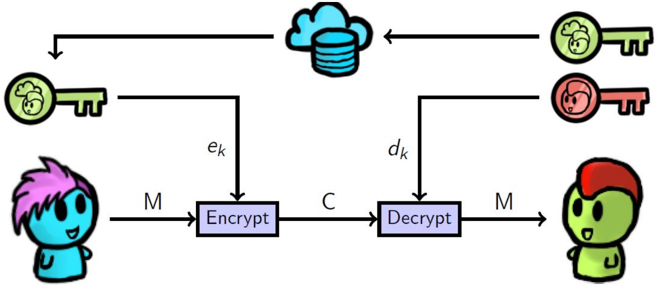

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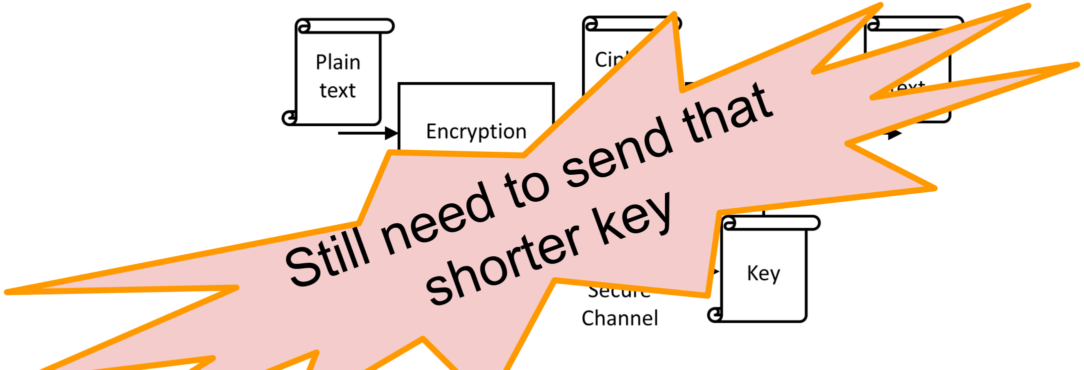

1. Shorter keys
2. Faster
3. Same key for E(m) and D(c)

Hybrid Cryptography

- Combine the two!!!!!!!
- Pick a random “128-bit” key K for a secret-key system
- Encrypt the large message with the key K (e.g., using AES)

And then…

- Encrypt the key K using a public-key system!
- Send the encrypted message and encrypted key to Bob

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Hybrid cryptography is used in (many) applications on the internet

Just Checking...

Public: (e_A, d_A)

Secret: K

Public: (e_B, d_B)

Secret: ?

- Enc/Dec functions: $E_{key}(*)$, $D_{key}(*)$
- Alice wants to send a **large** message *m* to Bob,

Q: How should Alice build the message efficiently? How does Bob recover m?

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Public: (e_A, d_A)

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Public: (e_B, d_B)

Secret: ?

- Enc/Dec functions: $E_{key}(*)$, $D_{key}(*)$
- Alice wants to send a **large** message *m* to Bob,

Q: How should Alice build the message efficiently? How does Bob recover m?

A: Alice computes $c_1 = E_{eb}(K)$, $c_2 = E_K(m)$ and sends $<c_1||c_2>$
Bob recovers $K = D_{db}(c_1)$ and then $m = D_K(c_2)$

Thursday: More Cryptography…